Name of College S.S. college, Jehanabad
Deptt - Mathematics
Topic - St. line (Analytical Geometry of 3-D)
Class - B.Sc I (Hons + Sub)
Time - 1.00 P.M to 1.45 P.M Date - 10-08-2020

PAGE NO 1
DATE 10-08-2020

By - Dr. Shree Nath Sharma.

Study Material - A st. line may be completely determined

(i) When we know its direction ~~cosines~~ and the co-ordinate of any point on it.
[Since we know from a given direction and a point one and only st. line may be drawn and from two given points one and only one st. line can be drawn.]

or

(ii) When any two points on the st. line are given.

(iii) A st. line may also be determined as the intersection of two planes.

(iv) Two equations of the first degree in x, y, z represent a st. line.

Symmetric Form → Find the Equation of st. line passing through a given point (α, β, γ) and whose direction cosines are l, m, n.

Let $A(\alpha, \beta, \gamma)$ be a given point.
Let $P(x, y, z)$ be any point on it.
Let $AP = r$
On projecting AP on the

Co-ordinate axes are.

$x-\alpha = lr \quad y-\beta = mr \quad z-\gamma = nr$

$\Rightarrow \frac{x-\alpha}{l} = r \quad \frac{y-\beta}{m} = \quad \frac{z-\gamma}{n} = r$

$\therefore \frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} = r$ is the required Equation of st-line

Remarks $\rightarrow$ $x = lr + \alpha$, $y = mr + \beta$, $z = nr + \gamma$ | be the General Co-ordinates of any Point on the st. line at a distance r.

(2) Equation of st. line passing through two given Points (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Problem (3) Method For the Finding the Equation of st. line of intersection of the planes $a_1x+b_1y+c_1z+d_1=0$, $a_2x+b_2y+c_2z+d_2=0$ in Symmetrical Form.

Let $a_1x+b_1y+c_1z+d_1=0$

$a_2x+b_2y+c_2z+d_2=0$

be two given planes

Let us Take two plane through the origin and Parallel to the given planes and their Equations are

$$a_1 x + b_1 y + c_1 z = 0 \quad \text{---} \; (I)$$
$$a_2 x + b_2 y + c_2 z = 0 \quad \text{---} \; (II)$$

Clearly the line of intersection of these two new planes is parallel to the original line

Solving (1) and (11) using Cross multiplication

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{y}{a_2 c_1 - a_1 c_2} = \frac{z}{a_1 b_2 - a_2 b_1}$$

thus Direction ratios of the given line are

$$b_1 c_2 - b_2 c_1, \quad c_1 a_2 - a_1 c_2, \quad a_1 b_2 - a_2 b_1$$

for Equation of line in Symmetrical form, we should know the co-ordinate of any point through which it passes

Now let the point chosen be ~~the~~ the one where the given line meets the plane $z = 0$

$\therefore$ $a_1x + b_1y + d_1 = 0$

$a_2x + b_2y + d_2 = 0$

By cross multiplication

$$\frac{x}{b_1d_2 - b_2d_1} = \frac{y}{d_1a_2 - a_1d_2} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\therefore x = \frac{b_1d_2 - b_2d_1}{a_1b_2 - a_2b_1} \quad y = \frac{d_1a_2 - a_1d_2}{a_1b_2 - a_2b_1}$$

$\therefore \left(\frac{b_1d_2 - b_2d_1}{a_1b_2 - a_2b}, \frac{d_1a_2 - a_1d_2}{a_1b_2 - a_2b_1}, 0 \right)$ is a point on the line hence

the symmetrical form is
using the formula

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\frac{x - \frac{b_1d_2 - b_2d_1}{a_1b_2 - a_2b_1}}{b_1c_2 - b_2c_1} = \frac{y - \frac{d_1a_2 - a_1d_2}{a_1b_2 - a_2b_1}}{c_1a_2 - a_1c_2} = \frac{z - 0}{a_1b_2 - a_2b_1}$$

3 Conditions that the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ may be wholly in the plane $ax+by+cz+d=0$

Sol^n. The Given line is

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} = r \text{ (say)}$$

$$\Rightarrow x = lr+\alpha$$
$$y = mr+\beta$$
$$z = nr+\gamma$$

The st. line would lie on the plane if these co-ordinates satisfy the Equation of the plane for all values of r

$$\therefore a(lr+\alpha) + b(\beta+mr) + c(\gamma+nr) + d = 0$$

$$\Rightarrow r(al+bm+cn) + (a\alpha+b\beta+c\gamma+d) = 0$$

This relation is true for every value of r.

$$\therefore al+bm+cn = 0$$
$$a\alpha+b\beta+c\gamma+d = 0$$

These are the required conditions